Quantum stochastic walks

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About IITiS PAN

About IITiS PAN
Where? Who? What?

About IITiS PAN

Where? Who? What?

Employment: 38 researchers on different levels.

Research in the are of computer science (theoretical informatics) and computer engineering (applied informatics):

- theoretical – quantum information;
- applied – machine learning, computer networks.

Current project involvement:

- coordinator for one H2020 project;
- coordinator for 4 Polish national projects;
- partner in 3 national projects;
- MC member in 4 COST networks.

Intro to quantum computing

Intro to quantum computing
Rules of quantum computing

Encoding classical data

- Use **vectors** as states, *eg.* using binary representation $0 \to |0\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}, \quad 1 \to |1\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix}, \quad 2 \to |2\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}.$
- Sum of two states is also **a state** ($\equiv$ quantum superposition), *eg.* $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\end{pmatrix}$.

Intro to quantum computing
Rules of quantum computing

Decoding quantum states

- Base vectors (states) describe possible classical data we can readout from the system.
- We use normalized states to interpret $(|\langle 0|x\rangle|^2, |\langle 1|x\rangle|^2, |\langle 2|x\rangle|^2)$ as a probability distribution.

Intro to quantum computing
States and operations

- Conservation of energy $\rightarrow$ the evolution has to be reversible.
- Length of the vector has to be preserved $\rightarrow$ evolution is unitary (or anti-unitary).

Intro to quantum computing

States and operations

- There are only two reversible operations on one bit: identity and negation.
- Given a reversible classical operation, its quantum counterpart is defined by its application on the base states $|0\rangle$ and $|1\rangle$.

For operations acting on one bit, the simplest example is the binary negation (NOT) operation. Its quantum counterpart is given by a matrix which simply interchanges the base states,

$$NOT = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Intro to quantum computing
States and operations

From this we can easily get a non-classical gate – the $\sqrt{NOT}$ gate

$$\sqrt{NOT} = \frac{1}{2}\begin{pmatrix} i+1 & i-1 \\ i-1 & i+1 \end{pmatrix},$$

which results in states "half way" between the input state and the negated input state.

$$\sqrt{NOT}|0\rangle = \frac{1+i}{2}|0\rangle + \frac{1-i}{2}|1\rangle,$$
$$\sqrt{NOT}|1\rangle = \frac{1-i}{2}|0\rangle + \frac{1+i}{2}|1\rangle.$$

Intro to quantum computing
Algorithms

Two basic ingredients of all quantum algorithms:

- Quantum Amplitude Amplification $\rightarrow$ unstructured searching;
- Quantum Fourier Transform $\rightarrow$ prime factorization.

Intro to quantum computing
Programming

- *The problem is how best to program these devices. The stakes are high get this wrong and we will have experiments that nobody can use instead of technology that can change the world.*
- *It is crucial that research on quantum-computing algorithms is tied more closely to research on the software that's used to implement them.*

Source: W. Zeng *et al.*, *First quantum computers need smart software*, Nature 549, 149151 (14 September 2017)

Rigetti Forest – a suite of tools for experimental quantum programming http://www.rigetti.com/forest

Intro to quantum computing

So what?

Quantum mechanics gives us rich set of allowed states and operations. But we do fully understand how to use them.

Quantum walks

Quantum walks

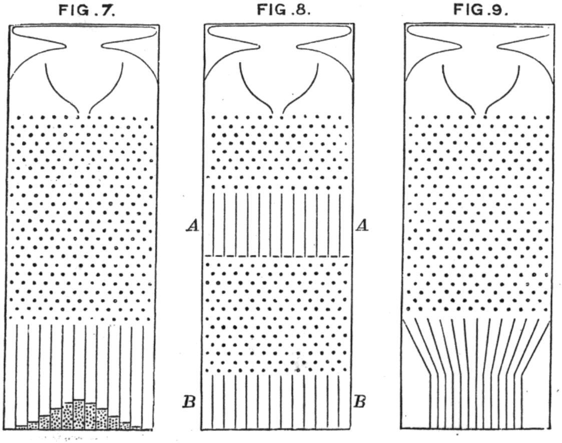
Quantum walks?

Quantum walks are useful for

- porting classical concepts, especially related to graph exploration, to the quantum domain;
- modeling quantum networks (*ie.* systems of connected processing units);
- studying non-classical (and non-quantum) properties of physical theories.

From random walks to quantum walks

Simple model of random walk



(F. Galton, Natural Inheritance, Macmillan, 1889, p. 63. See: http://galton.org/)

Quantum walks
Galton board

Quantum evolution must be linear and must preserve probabilities. This is reflected in the fact that quantum gates are unitary and thus reversible.
The direct counterpart of classical evolution of the Galton board encoded in quantum states reads

$$|x-1\rangle \mapsto \alpha|x-2\rangle + \beta|x\rangle$$
$$|x+1\rangle \mapsto \alpha|x\rangle + \beta|x+2\rangle$$

The initial states are orthogonal, $\langle x-1|x+1\rangle = 0$.

Quantum coins

One needs a coin in order to fully specify quantum evolution.

Quantum walks
Quantization of random walks

The basic model of quantum walk is described as

$$U = S(C \otimes \mathbb{1})$$

where S (shift) describes a step of the walker, and C (coin) governs the choice of direction. For graph of degree 2, shift is defined as a controlled operation of the form

$$\sum_x |0\rangle\langle 0| \otimes |x-1\rangle\langle x| + |1\rangle\langle 1| \otimes |x+1\rangle\langle x|.$$

Quantum walks
Quantization of random walks

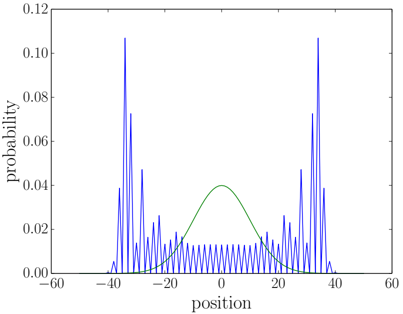
Decoding quantum information

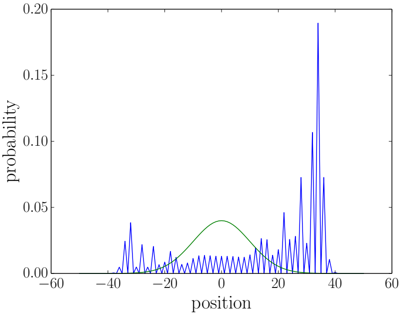
As the final step of each quantum algorithm one has to perform a measurement to decode the information and make it useful for the classical world.

Quantum walks...

... are not random – the state is completely described at each time step.

Quantum walks
Quantization of random walks





Evolution on the line governed by a quantum walk with a symmetric coin (blue line). In the first case the initial state of the coin register is $|0\rangle + i|1\rangle$ and in the second case it reads $|0\rangle$.

Stochastic quantum walks

Stochastic quantum walks

How we can map graphs onto quantum walks?

- The unitary evolution can be described by $U = \exp(tH)$, with t representing the time of evolution.
- Matrix H can be taken as an adjacency matrix of a graph.
- This enables us to
  - describe unitary process of quantum walk between levels representing graph edges;
  - incorporate the information about the interaction with the environment.

Stochastic quantum walks

- By introducing interaction with the environment we loose determinism (and reversibility).
- Now we have to describe the stat using probabilities of getting vector states (mixed states ≡ probability distribution on the vectors states).
- Bu we can explore directed graphs using quantum speedup.

Stochastic quantum walks

Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) master equation describes general evolution of quantum states.

$$\frac{\mathrm{d}}{\mathrm{d}t}\varrho = -\mathrm{i}(1-\omega)[H,\varrho] + \omega \sum_{L\in\mathcal{L}} \left(L\varrho L^\dagger - \frac{1}{2}\{L^\dagger L, \varrho\} \right)$$

- Hamiltonian H describes the evolution of the closed system,
- Lindblad operators $\mathcal{L}$ describe the evolution of the open system.
- Value $\omega = 0$ means we have only the unitary ($\equiv$ reversible) part.

Stochastic quantum walks

Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) master equation describes general evolution of quantum states.

$$\frac{\mathrm{d}}{\mathrm{d}t}\varrho = (1-\omega)f_1(H,\varrho) + \omega f_2(\mathcal{L},\varrho)$$

- Hamiltonian H describes the evolution of the closed system,
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Stochastic quantum walks

- For time-independent H and $\mathcal{L}$ we can solve the equation analytically.
- The resulting evolution is given by

$$\exp\left(tF(H,\mathcal{L})\right).$$

where F is a linear form of H and elements of the elements of $\mathcal{L}$.

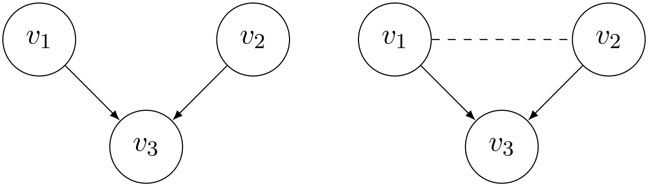
Stochastic quantum walks

Spontaneous moralization on directed graphs

The addition of the quantum (≡ reversible) part makes the process reversible and thus unsuitable for using on directed graphs.

Stochastic vs directed

In the global interaction regime process is reversible (and thus unsuitable for using on directed graphs) even without the quantum part .



Stochastic quantum walks
Digraph structure observance

Assume that from each vertex there is path to some sink vertex.

Digraph structure observance

Arbitrary initial state converges to the state spanned by vectors corresponding to the sink vertices from the condensation of the graph.

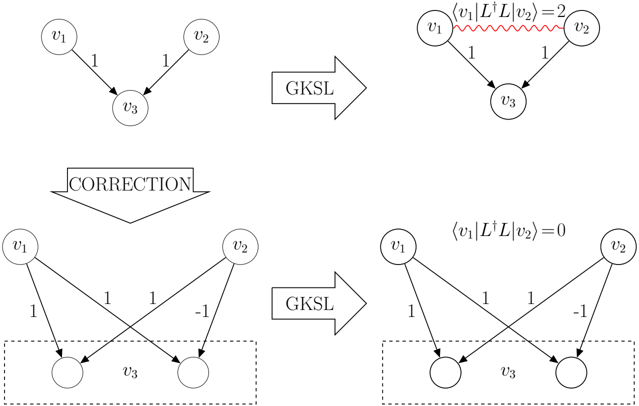
In the case of the example arbitrary state should converge to the state ϱ_{v_3}. However, one can find that

$$\frac{1}{4}(|v_1\rangle - |v_2\rangle)(\langle v_1| - \langle v_2|) + \frac{1}{2}|v_3\rangle\langle v_3|$$

is proper stationary state.

Stochastic quantum walks
Digraph structure observance

Solution: extend the state space.



Stochastic quantum walks
Digraph structure observance

Numerical analysis of the digraph structure observance

- Start in some vertex with nonzero outdegree.
- Determine the state ϱ_∞ for large t.
- Verify whether it is close to stationary state in the sense of probability distribution of measurement.

Stochastic quantum walks
Digraph structure observance

Cumulated probability of measuring the state in the sink vertices

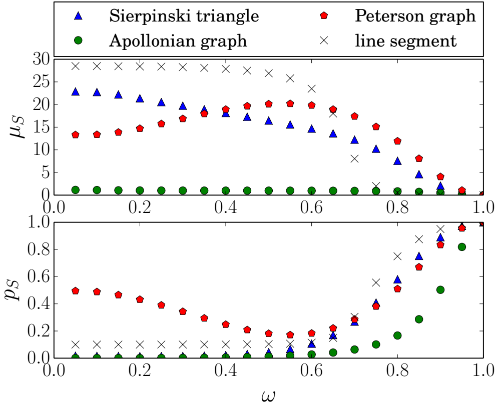
$$p_S(\varrho_\infty) = \sum_{v \in \bigcup S(\mathbb{G})} \langle v|\varrho_\infty|v\rangle$$

and the second moment of the distance from the sink vertices

$$\mu_S(\varrho_\infty) = \sum_{v \in V} d^2(v, S)\langle v|\varrho_\infty|v\rangle.$$

Stochastic quantum walks

Digraph structure observance

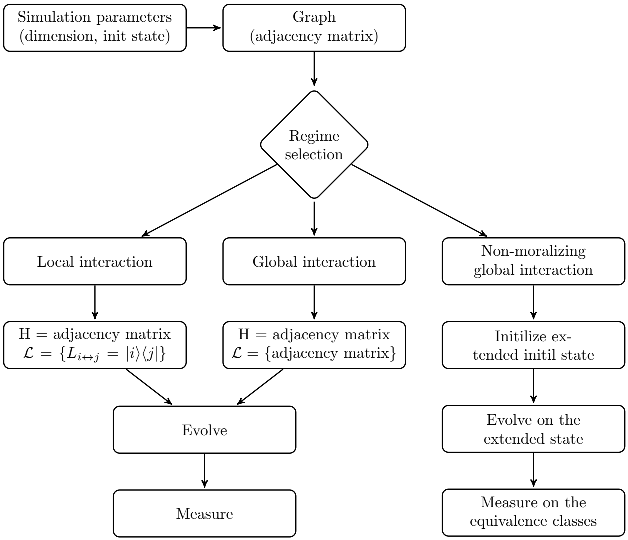


QSWalk.jl package

Why Julia?

- Interactive development.
- Numerical capabilities.
- Parallel computing.

QSWalk.jl package



QSWalk.jl package

(example in Jupyter)

Acknowledgements

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Thank you!

https://iitis.pl/ miszczak

https://github.com/ZKSI/QSWalk.jl

https://quantiki.org/wiki/list-qc-simulators