States and channels in quantum mechanics without complex numbers

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In this work we demonstrate a simplified version of quantum mechanics in which the states are constructed using real numbers only. In the standard formulation of quantum mechanics the state is represented by positive semidefinite, normalized linear operators. In the following we focus on linearity and hermicity properties of density matrices as they are crucial for the properties of quantum channels.

The main advantage of the introduced approach is that the simulation of the n-dimensional quantum system requires $\mathscr{O}(n^2)$ real numbers, in contrast to the standard case where $\mathscr{O}(n^4)$ real numbers are required.

The main disadvantage is the lack of hermicity in the representation of quantum states. This leads to the occurrence of complex eigenvalues of the real-valued density matrices.

During the last few years *Mathematica* computing system has become very popular in the area of quantum information theory and the foundations of quantum mechanics. The main reason for this is its ability to merge the symbolic and numerical capabilities, both of which are often necessary to understand the theoretical and practical aspects of quantum mechanical systems [1, 2, 3].

We utilize the symbolic calculation capabilities offered by *Mathematica* [4] to investigate the properties of the variant of quantum theory based of the representation of density matrices built using real-numbers only. We develop a set of functions for manipulating real quantum states. With the help of this tool we study the properties of the introduced representation and the induced representation of quantum channels.

We start by introducing the said representation, including the required functions. Next we test the behavior of selected partial operations in this representation and we consider the general case of quantum channels acting on the space of real density matrices. In the last part we provide some insights into the spectral properties of the real density matrices. Finally, we provide the summary and the concluding remarks.

References

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