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Combined Diffusion Approximation - Simulation model of AQM's transient behaviour

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ABSTRACT

The article introduces an approach combining diffusion approximation and simulation ones. Furthermore, it describes how it can be used to evaluate active queue management (AQM) mechanisms. Based on the obtained queue distributions, the simulation part of the model decides on package losses and modifies the flow intensity sent by the transmitter. The diffusion is used to estimate queue distributions and the goal of the simulation part of the model is to represent the AQM mechanism. On the one hand, the use of the diffusion part considerably accelerates the performance of the whole model. On the other hand, the simulation increases the accuracy of the diffusion part. We apply the model to compare the performance of fractional order PI^η controller used in AQM with the performance of RED, a well known active queue management mechanism.

1. Introduction

The efficiency of the TCP protocol depends mainly on the queue management algorithm used in a router. There are distinguished two fundamental principles of network packet management. In passive queue management, packets coming to a buffer are rejected only if the buffer is fully occupied. The active mechanisms are based on the preventive packet dropping when there is still a place to store them in the queue. The packets are dropped randomly following the assumed probability loss function. This approach enhances the throughput and fairness of the link-sharing and eliminates global synchronization. The primary active queue management algorithm (AQM) is Random Early Detection (RED), primarily proposed in 1993 by Sally Floyd and Van Jacobson [1]. Several works studied the impact of various parameters on the RED performance [2, 3], and many variations of AQM mechanism were developed to improve its performance, e.g. [4, 5]. In order to evaluate the AQMs' performance properly, it is necessary one create realistic models of them.

1.1. Our contribution

The aim of this article is to create a model combining diffusion approximation and simulation. Such a model enables us to take advantages of both approaches. Diffusion approximation is suitable to analyze transient states and therefore

accelerates the performance of the whole model. Including the simulation in the model will increase the accuracy of the diffusion part. The proposed model of a TCP connection having a bottleneck router with AQM policy is a combination of diffusion and simulation model. We use diffusion approximation to estimate the distribution of the queue length. Based on the distribution, the average queue length is calculated.

Depending on the computed average queue length, the simulation part of the model decides the packet rejection. In the case of active queue management, the decision of dropping packets is random and depends on queue occupancy and dropping packet probability function. The decision of packet rejection reduces by half the intensity λ of the traffic source. In the absence of a loss, the intensity of the source increases linearly. This mechanism may be seen as a part of the closed-loop control of TCP/IP traffic intensity. The evolution of the source intensity can be illustrated as follows:

$$\lambda = \begin{cases} \lambda + \zeta & \text{if} \quad AQM \ decides \ no \ loss \\ \frac{\lambda}{2} & \text{if} \quad AQM \ decides \ a \ loss \end{cases} \quad (1)$$

where ζ is assumed a constant increase in the intensity of the source. Such source behaviour is a simplified model of the TCP NewReno algorithm [6].

Using our combined model, we investigate the behaviour of fractional order controller PI^η applied to control Internet traffic supervised by TCP transport protocol. We also investigate the influence of parameters of the controller on the queue length and evolution of the TCP congestion window. The performance of fractional order PI^η controller is compared to the performance of RED.

The remainder of the paper is organized as follows: Section 2 reminds works related to this article. Section 3 summarizes the basics of the diffusion approximation. In this section we present two used networks station models:

unlimited queue: G/G/1 and limited queue: G/G/1/N. Section 4 presents theoretical bases for PI^η controller, next used in proposed combined diffusion approximation-simulation model. Section 5 discusses the obtained results. Concluding remarks are presented in section 6.

2. Related works

The diffusion approximation has many practical applications including the modeling of financial market [7]. It has been popular as a tool for road congestion analysis [8]. It is useful, for example, in risk assessments of sea turtle populations [9]. This method has been used in the evaluation of computer systems and networks for many years, see, e.g., the bibliography in [10], a more recent discussion of the errors introduced by the method in case of G/G/1 queue applied to a traffic server performance is presented in [11]. It may also be used for other than FIFO queueing disciplines, e.g., priority [12, 13]. The alternative approach of transient analysis, which may be applied in traffic control models [14, 15] is fluid flow approximation [16], simpler and concerning only mean values of queues.

The solution proposed in this paper is also based on diffusion approximation and G/G/1/N queuing model like in [17]. The increase in the number of users with permanent access to the Internet has created renewed interest in the research of multi-server queuing models. In the article [18] the limits of the GI/M/n/∞ were studied.

The initial AQM mechanism RED (Random Early Detection) was proposed by IETF and was primarily described by Sally Floyd and Van Jacobson in [1]. It is based on a drop function giving the probability that a packet is rejected. The argument of this function is a weighted moving average queue length, acting as a low-pass filter. This average depends on actual queue occupancy and a previously calculated value of the weighted moving average. The packet dropping probability is based on a linear function.

The introduction of AQM mechanisms has significantly improved the quality of network transmission. However, RED has such drawbacks as low throughput, unfair bandwidth sharing, the introduction of variable latency, deterioration of network stability [19]. For these reasons, numerous propositions of improvements appeared. One of these modifications is DSRED (double-slope RED) introduced and developed in [20, 21]. This solution resolves the linear packet dropping function by the drop function composed of two lines with different slopes. The paper [22] proposes the NLRED algorithm with a quadratic dropping function. In the paper [23], authors considered the polynomial function.

The article [24] describes a Proportional-Integral controller on the low-frequency dynamics. The paper [25] described a robust controller, based on a known technique for H^∞ control of systems with time delays. The PI AQM controller proposed in [24] was designed following the small-gain theorem. Easy implementation of PI AQM controllers in real networks resulted in a number of propositions [26, 27, 28, 29]. The first application of the fractional order PI^η

controller as an AQM policy in a fluid flow model of a TCP connection was presented in [30].

Article [31] introduces a simple diffusion model of a RED control mechanism working in an open-loop scenario. Our paper [32] describes diffusion models of a RED and a fractional order PI^η controller working under TCP NewReno's control. To the best of our knowledge, there is no paper describing a combination of diffusion-simulation approaches.

3. Diffusion approximation

The main principle of the diffusion approximation method [33, 34, 35, 36], is replacing the discrete process $N(t)$ i.e. the number of customers in the queue, by a continuous diffusion process $X(t)$. Similarly as in the case of $N(t)$, the changes of $dX(t) = X(t + dt) - X(t)$ are normally distributed with the mean βdt and variance αdt determined by the parameters β, α of the diffusion equation [37, 38]:

$$\frac{\partial f(x,t;x_0)}{\partial t} = \frac{\alpha}{2}\frac{\partial^2 f(x,t;x_0)}{\partial x^2} - \beta\frac{\partial f(x,t;x_0)}{\partial x} \quad (2)$$

where $f(x,t:x_0)$ is the probability density function (pdf) of the diffusion process [38]:

$$f(x,t;x_0)\,dx = P[x \le X(t) < x+dx|X(0) = x_0]. \quad (3)$$

The solution of Eq. (2) is used for the evaluation of the queue distribution in the investigated. It may be applied in case when the interarrival and service times follow general distributions $A(x)$, $B(x)$, i.e. in modelling G/G/1 and G/G/1/N service systems. Two first moments of these distributions are considered: $E[A] = 1/\lambda$, $E[B] = 1/\mu$, $Var[A] = \sigma_A^2$, $Var[B] = \sigma_B^2$.
Denote also the squared coefficients of variation: $C_A^2 = \sigma_A^2\lambda^2$, $C_B^2 = \sigma_B^2\mu^2$. The choice of diffusion parameters:

$$\beta = \lambda - \mu, \quad \alpha = \sigma_A^2\lambda^3 + \sigma_B^2\mu^3 = C_A^2\lambda + C_B^2\mu \quad (4)$$

3.1. Unlimited queue: G/G/1 station

More formal justification of diffusion approximation is in limit theorems for $G/G/1$ system given by Iglehart and White [39, 40, 41, 42]: If $\hat{N}_n$ is a series of random variables derived from $N(t)$ [10]:

$$\hat{N}_n = \frac{N(nt) - (\lambda - \mu)nt}{(\sigma_A^2\lambda^3 + \sigma_B^2\mu^3)\sqrt{n}} \quad (5)$$

then the series is weakly convergent (in the sense of distribution) to ξ where $\xi(t)$ is a standard Wiener process (i.e. diffusion process with $\beta = 0$ i $\alpha = 1$) provided that $\varrho > 1$, that means if the system is overloaded and has no equilibrium state. In the case of $\varrho = 1$ the series $\hat{N}_n$ is convergent to ξ_R. The $\xi_R(t)$ process is $\xi(t)$ process limited to half-axis $x > 0$ [42]:

$$\xi_R(t) = \xi(t) - \inf[\xi(u), \ 0 \le u \le t]. \quad (6)$$

The process $N(t)$ is never negative, therefore $X(t)$ should also be limited to values $x \ge 0$. It is done by placing at

$x = 0$ a barrier which prevents the process from moving into negative part of x axis.

One choice is to place at $x = 0$ a *reflecting barrier* [43] that limits the process to a positive x-axis and is equivalent to the condition:

$$\int_0^\infty f(x, t; x_0)dx = 1$$

and:

$$\frac{\partial}{\partial t} \int_0^\infty f(x, t; x_0)dx = \int_0^\infty \frac{\partial f(x, t; x_0)}{\partial t} dx = 0. \quad (7)$$

Replacing the integrated function with the right side of the diffusion equation, we get the boundary condition corresponding to the reflecting barrier at zero [10]:

$$\lim_{x \to 0} [\frac{\alpha}{2} \frac{\partial f(x, t; x_0)}{\partial x} - \beta f(x, t; x_0)] = 0. \quad (8)$$

The solution of Eq. (3) with boundary conditions defined by Eq. (8) gives us: [43]

$$f(x, t; x_0) = \frac{\partial}{\partial x} [\Phi(\frac{x - x_0 - \beta t}{\alpha t}) - e^{\frac{2\beta x}{\alpha}} \Phi(\frac{x + x_0 + \beta t}{\alpha t})], \quad (9)$$

where: $\Phi(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ is the PDF of standard normal distribution.

For $(\beta < 0)$ the system converges to a steady-state: $\lim_{t \to \infty} f(x, t; x_0) = f(x)$ and partial differential equation (2) becomes an ordinary one [10]:

$$0 = \frac{\alpha}{2} \frac{d^2 f(x)}{dx^2} - \beta \frac{d f(x)}{dx} \quad (10)$$

with solution:

$$f(x) = -\frac{2\beta}{\alpha} e^{\frac{2\beta x}{\alpha}}.$$

This formula approximates the queue at $G/G/1$ system [10]:

$$p(n, t; n_0) \approx f(n, t; n_0), \quad (11)$$

and at steady-state $p(n) \approx f(n)$; one can also choose e.g. [37]

$$p(0) \approx \int_0^{0.5} f(x)dx, p(n) \approx \int_{n-0.5}^{n+0.5} f(x), n = 1, 2, \dots, \quad (12)$$

The reflecting barrier in point $x = 0$ causes the immediate reflecting of the process, that means $f(0, t; x_0) = 0$. This version of the diffusion process is a heavy-load approximation and gives good results in case of a system with utilization factor ρ close to 1, i.e. probability of the empty system is closed to 0.

This restriction is removed by another type of limit condition at $x = 0$: a *barrier with instantaneous elementary returns* (sometimes called jumps) [38]. When the diffusion process reaches $x = 0$, it remains there for a random time corresponding to the idle period with no customers in the queue and the returns to $x = 1$ (it corresponds to the arrival of one packet to the system).
The time when the process is at $x = 0$ means that the system is in the idle state. The diffusion equation becomes [38]:

$$\frac{\partial f(x, t; x_0)}{\partial t} = \frac{\alpha}{2} \frac{\partial^2 f(x, t; x_0)}{\partial x^2}$$
$$-\beta \frac{\partial f(x, t; x_0)}{\partial x} + \lambda p_0(t)\delta(x - 1),$$

$$\frac{dp_0(t)}{dt} = \lim_{x \to 0} [\frac{\alpha}{2} \frac{\partial f(x, t; x_0)}{\partial x} - \beta f(x, t; x_0)] - \lambda p_0(t), \quad (13)$$

where: $p_0(t) = P[X(t) = 0]$.

The term $\lambda p_0(t)\delta(x-1)$ gives the probability density that the process is started at point $x = 1$ at the moment t, because of the jump from the barrier. The second equation balances $p_0(t)$: the term $\lim_{x \to 0} [\frac{\alpha}{2} \frac{\partial f(x, t, x_0)}{\partial x} - \beta f(x, t; x_0)]$ gives the probability flow *into* the barrier, while $\lambda p_0(t)$ represents the probability flow *out* of the barrier [10].

The approach to obtain the function $f(x, t; x_0)$ of the process with jumps from the barrier is to express it with the use of another pdf $\phi(x, t; x_0)$ for the diffusion process with the absorbing barrier at $x = 0$ [17]. This process starts at $t = 0$ from $x = x_0$ and ends when it reaches the barrier. Such probability density function is easier to determine [44]:

$$\phi(x, t; x_0) = \frac{e^{\frac{\beta}{\alpha}(x-x_0) - \frac{\beta^2}{2\alpha}t}}{\sqrt{2\pi\alpha t}} [e^{-\frac{(x-x_0)^2}{2\alpha t}} - e^{-\frac{(x+x_0)^2}{2\alpha t}}] \quad (14)$$

The density function of the first passage time from $x = x_0$ to $x = 0$ is [17]:

$$\gamma_{x_0,0}(t) = \lim_{x \to 0} [\frac{\alpha}{2} \frac{\partial}{\partial x} \phi(x, t; x_0) - \beta\phi(x, t; x_0)] =$$

$$= \frac{x_0}{\sqrt{2\pi\alpha t^3}} e^{-\frac{(\beta t + x_0)^2}{2\alpha t}} \quad (15)$$

Assume that process with the barrier with jumps starts at $t = 0$ at a point $x > 0$ with density $\psi(x)$ and when it reaches the barrier, it stays there for a time defined by a density function $l_0(x)$, then jumps to the point $x = 1$ and again, after certain time, returns to $x = 0$, stays there, jumps to $x = 1$, etc. The total stream $\gamma_0(t)$ of probability mass that enters the barrier is [17]:

$$\gamma_0(t) = p_0(0)\delta(t) + [1 - p_0(0)]\gamma_{\psi,0}(t) + \int_0^t g_1(\tau)\gamma_{1,0}(t-\tau)d\tau, \quad (16)$$

where:

$$\gamma_{\psi,0}(t) = \int_0^\infty \gamma_{\xi,0}(t)\psi(\xi)d\xi,$$

$$g_1(\tau) = \int_0^\tau \gamma_0(t)l_0(\tau - t)dt. \qquad (17)$$

The density function of the diffusion process with instantaneous returns is [17]:

$$f(x, t; x_0) = \phi(x, t; \psi) + \int_0^t g_1(\tau)\phi(x, t-\tau; 1)d\tau. \quad (18)$$

Denote by $\overline{f}(s)$ the Laplace transform of a function $f(x)$. For the Eqs. (16) and (17) there are following Laplace transforms [17]:

$$\overline{\gamma}_0(s) = p_0(0) + [1 - p_0(0)]\overline{\gamma}_{\psi,0}(s) + \overline{g}_1(s)\overline{\gamma}_{1,0}(s),$$

$$\overline{g}_1(s) = \overline{\gamma}_0(s)\overline{l}_0(s), \qquad (19)$$

where:

$$\overline{\gamma}_{x_0,0}(s) = e^{-x_0\frac{\beta+A(s)}{\alpha}}, \ A(s) = \sqrt{\beta^2 + 2\alpha s},$$

$$\overline{\gamma}_{\psi,0}(s) = \int_0^\infty \overline{\gamma}_{\xi,0}(s)\psi(\xi)d\xi, \qquad (20)$$

and then:

$$\overline{g}_1(s) = [p_0(0) + [1 - p_0(0)]\overline{\gamma}_{\psi,0}(s)]\frac{\overline{l}_0(s)}{1 - \overline{l}_0(s)\overline{\gamma}_{1,0}(s)}. \quad (21)$$

Equation (18) in terms of the Laplace transform becomes [17]:

$$\overline{f}(x, s; x_0) = \overline{\phi}(x, s; \psi) + \overline{g}_1(s)\overline{\phi}(x, s; 1), \quad (22)$$

where:

$$\overline{\phi}(x, s; x_0) = \frac{e^{\beta(x_C x_0)}}{A(s)}[e^{-|x-x_0|\frac{A(s)}{\alpha}} - e^{-|x+x_0|\frac{A(s)}{\alpha}}],$$

$$\overline{\phi}(x, s; \psi) = \int_0^\infty \overline{\phi}(x, s; \xi)\psi(\xi)d\xi. \qquad (23)$$

The inverse transforms of these functions can be obtained numerically. For this purpose, Stehfest's algorithm [45] is used.

The presented above G/G/1 model assumes constant model parameters. In the case of variable model parameters, we should divide the model time into periods, where the parameters of the model are constant. In this approach the results obtained at the end of one time period serve as the initial condition for the next period.

3.2. Limited queue: G/G/1/N station

In the case of G/G/1/N station, the number of packets in the node is limited to N. That means that we need in our model the second barrier placed at $x = N$.

So in G/G/1/N model, the diffusion process is limited by two barriers. The first barrier is placed in $x = 0$, and the second is placed at $x = N$. The behaviour of the process in the first barrier we described in the previous subsection. In the second barrier, when the process reaches $x = N$, it waits there for a time corresponding to the period when the queue is full and incoming packets are being lost and then jumps to $x = N - 1$. The density function of such process $f(x, t; x_0)$ is obtained similarly to the model with infinite queue described before.

In the first step, the method determines the density $\phi(x, t; x_0)$ of the diffusion process with two absorbing barriers at $x = 0$ and $x = N$, started at $t = 0$ from $x = x_0$ [44]:

$$\phi(x, t; x_0) = \frac{1}{\sqrt{2\pi\alpha t}}\sum_{n=-\infty}^{\infty}(\exp[\frac{\beta x_n'}{\alpha} - \frac{(x - x_0 - x_n' - \beta t)^2}{2\alpha t}]$$

$$-\exp[\frac{\beta x_n''}{\alpha} - \frac{(x - x_0 - x_n'' - \beta t)^2}{2\alpha t}]), \qquad (24)$$

where: $x_n' = 2nN, x_n'' = -2x_0 - x_n'.$

For the initial condition is defined by a function $\psi(x)$, $x \in (0, N)$, $\lim_{x\to 0}\psi(x) = \lim_{x\to N}\psi(x) = 0$, then the pdf of the process has the form:

$$\phi(x, t; \psi) = \int_0^N \phi(x, t; \xi)\psi(\xi)d\xi. \qquad (25)$$

The density function $f(x, t; \psi)$ of diffusion process with instantaneous returns from both barriers is composed of function $\varphi(x, t; \psi)_1$ representing the influence of the initial condition ψ and the set of functions $\varphi(x, t - \tau; 1)$, $\varphi(x, t - \tau; N - 1)$ started after the jump from barrier at time $\tau < t$, at points $x = 1$ and $x = N - 1$ with intensities $g_1(\tau)$ and $g_{N-1}(\tau)$ [17]:

$$f(x, t; \psi) = \varphi(x, t; \psi) + \int_0^t g_1(\tau)\varphi(x, t - \tau; 1)d\tau$$

$$+ \int_0^t g_{N-1}(\tau)\varphi(x, t - \tau; N - 1)d\tau. \qquad (26)$$

Functions $g_1(t), g_N(t)$ can be expressed by means of probability densities $\gamma_0(t)$ and $\gamma_N(t)$ [31]:

$$g_1(\tau) = \int_0^\tau \gamma_0(t)l_0(\tau - t)dt,$$

$$g_{N-1}(\tau) = \int_0^\tau \gamma_N(t)l_N(\tau - t)dt, \qquad (27)$$

where: $l_0(x)$, $l_N(x)$ are density functions of the distribution of time which the process stays at $x = 0$ and $x = N$. Not that these distributions may be general and not restricted to exponential ones.

Probability densities $\gamma_0(t)$, $\gamma_N(t)$ that process enters the barrier at $x = 0$, or $x = N$ at time t are [17]:

$$\gamma_0(t) = p_0(0)\delta(t) + [1 - p_0(0) - p_N(0)]\gamma_{\psi,0}(t)$$
$$+ \int_0^t g_1(\tau)\gamma_{1,0}(t-\tau)d\tau + \int_0^t g_{N-1}(\tau)\gamma_{N-1,0}(t-\tau)d\tau,$$

$$\gamma_N(t) = p_N(0)\delta(t) + [1 - p_0(0) - p_N(0)]\gamma_{\psi,N}(t)$$
$$+ \int_0^t g_1(\tau)\gamma_{1,N}(t-\tau)d\tau + \int_0^t g_{N-1}(\tau)\gamma_{N-1,N}(t-\tau)d\tau,$$

$$(28)$$

where: $\gamma_{1,0}(t)$, $\gamma_{1,N}(t)$, $\gamma_{N-1,0}(t)$, $\gamma_{N-1,N}(t)$ are the densities of first passage times of diffusion process between corresponding points indicated in the lower index, [17]:

$$\gamma_{1,0}(t) = \lim_{x\to 0}\left[\frac{\alpha}{2}\frac{\partial\phi(x,t;1)}{\partial x} - \beta\phi(x,\,t;1)\right]. \quad (29)$$

Functions $\gamma_{\psi,0}(t)$, $\gamma_{\psi,N}(t)$ define probability densities, that the initial process, started on $t = 0$, at the point $x = \xi$ with density $\psi(\xi)$, will end at time t by entering the barrier respectively at $x = 0$ or $x = N$.

Finally, probabilities that at time t the process has value $x = 0$ or $x = N$ are:

$$p_0(t) = \int_0^t [\gamma_0(\tau) - g_1(\tau)]d\tau,$$

$$p_N(t) = \int_0^t [\gamma_N(\tau) - g_{N-1}(\tau)]d\tau. \quad (30)$$

Similarly to the previous model of the Subsect. 3.1, the presented equations are transformed using Laplace transform and determine the transformed function $\overline{f}(x, s; \psi)$, and it is original is obtained numerically. As previously, to vary with time the diffusion coefficients, we divide the time axis into subintervals with specific constant parameters, and the solution of the end of an interval gives initial conditions for the next one. The presented transient solution tends for $t \to \infty$, to the known steady-state solutions; therefore, models based on diffusion approximation can also refer to the steady-state. This approach was used in several models, e.g. in RED modelling [31], and its correctness was verified with simulations, e.g. [46].

4. AQM mechanism based on non-integer order PI^η controller

Recent scientific and engineering studies have shown that the dynamics of many systems can be described more precisely using non-integer differential equations [47]. Fractional Order Derivatives and Integrals (FOD/FOI) are a natural extension of the well-known integrals and derivatives.

Differintegrals of non-integer orders enable better and more precise control of dynamical processes allowing to include memory in a process. There are several definitions related to FOD/FOI. The most popular continuous definitions include: Riemann-Liouville [48], Atangana-Baleanu [49], Caputo–Fabrizio [50] and Liouville-Caputo [47]. There are also discrete solutions of Caputo definition [51] and Riemann-Liouville definitions [51, 52]. Our model takes into account discrete moments of packet arrivals so we decide to use Grünwald-Letnikov discrete fractional operator [53, 54].

A proportional-integral controller (PI controller) is a traditional mechanism used in feedback control systems. Earlier works [55, 56] indicates that the non-integer order controllers have better behaviour than classic ones. Authors of [57] confirm that fractional order controllers achieve better results than the best integer order controllers. There are many studies confirming the advantage of FOD over classical derivatives. The article [58] describes the use of fractional order PID controller to control the temperature in an isothermal room used in a pharmaceutical factory. The simulation results indicate that the proposed controller provides better quality of control than a classical PID controller with autotuning. The authors of the [59] also confirm that the non-integer PID controller provides better and more stable system operation than the traditional one.

The articles [60, 61, 62, 63], describe how to use the response of the PI^η(non-integer integral order) to calculate the response of the AQM mechanism. The probability of packet loss is given by the formula:

$$p_i = max\{0, -(K_P e_k + K_I \Delta^\eta e_k)\} \quad (31)$$

where K_P, K_I are tuning parameters, e_k is the error in current slot $e_k = Q_k - Q$, i.e. the difference between current queue Q_k and desired queue Q.

Thus, the dropping probability depends on three parameters: the coefficients for the proportional and integral terms K_p, K_i and integral order η.

In the active queue management, packet drop probabilities are determined at discrete moments of packet arrivals; hence the queue model should be considered as a case of discrete systems. We use Grunwald-Letnikov (GrLET) definition of the discrete differ-integrals of non-integer order. This definition is a generalization of the traditional definition of the difference of integer order to the non-integer order, and it is analogous to a generalization used in GrLET formula.

For a given sequence $f_0, f_1, ..., f_j, ..., f_k$ [64, 65]:

$$\triangle^\eta f_k = \sum_{j=0}^{k}(-1)^j\binom{\eta}{j}f_{k-j} \quad (32)$$

where $\eta \in R$ is generally a non-integer fractional order, f_k is a differentiated discrete function, and $\binom{\eta}{j}$ is generalized Newton symbol defined as follows:

$$\binom{\eta}{j} = \begin{cases} 1 & \text{for } j = 0 \\ \dfrac{\eta(\eta-1)(\eta-2)..(\eta-j+1)}{j!} & \text{for } j = 1, 2, ... \end{cases}$$

(33)

For $\eta = 1$ we get the formula for the difference of the first order (only two coefficients are non-zero).

$$\triangle^1 x_k = 1x_k - 1x_{k-1} + 0x_{k-2} + 0x_{k-3} \dots \quad (34)$$

For $\eta = -1$ we get the sum of all samples (the discrete integral of first order equivalent).

$$\triangle^{-1} x_k = 1x_k + 1x_{k-1} + 1x_{k-2} + 1x_{k-3} \dots \quad (35)$$

For non-integer derivative and integral order we get the weighted sum of all samples, e.g.:

$$\triangle^{-1.2} x_k = 1x_k + 1.2x_{k-1} + 1.32x_{k-2} + 1.408x_{k-3} \dots$$

$$\triangle^{-0.8} x_k = 1x_k + 0.8x_{k-1} + 0.72x_{k-2} + 0.672x_{k-3} \dots \quad (36)$$

5. Diffusion approximation analysis of AQM performance

The presented mixed diffusion-simulation model uses the limited queue, i.e. G/G/1/N station to obtain the time-dependent average queue length. Based on it, the simulation part of the model decides on dropping packets.

Calculations for both models (the mixed diffusion - simulation and simple simulation) were performed in Python and C. The simulations were done using the SimPy Python simulation packet. SimPy is released under the MIT License (free software license originating at the Massachusetts Institute of Technology). During the tests, we analyzed such parameters of the transmission with AQM as the length of the queue and changes of the source intensity λ.

The diffusion calculations were carried with $\Delta t = \frac{1}{\lambda}$ step. As a result, we obtain a queue distribution and then its mean size. On this basis, the simulation part of the model decides if the packet is rejected. The simulator of the AQM mechanism draws the value given by a uniform generator of the range <0;1>, compares it with the value of the packet dropping function (which is a function of the queue length) and decides if the packet is accepted or deleted. This decision influences the new value of λ; the change is made after Δt delay which represents the time after which the loss information is received by the TCP transmitter, and the traffic with new intensity reaches the router. The service time represents the time of a packet treatment and dispatching we assume it is exponentially distributed with constant parameter $\mu = 1$. The results of the mixed diffusion-simulation model are compared with pure simulation ones.

We considered two types of AQM algorithms: standard RED and algorithm based on the answer of PI^η controller.

The queue parameters were as follows: maximal queue lengths = 30, RED parameters $Min_{th} = 10$, $Max_{th} = 20$, PI^η desire queue length = 10.

To obtain reliable transient state results, we repeated the experiments 10,000 times. An example of the results of a
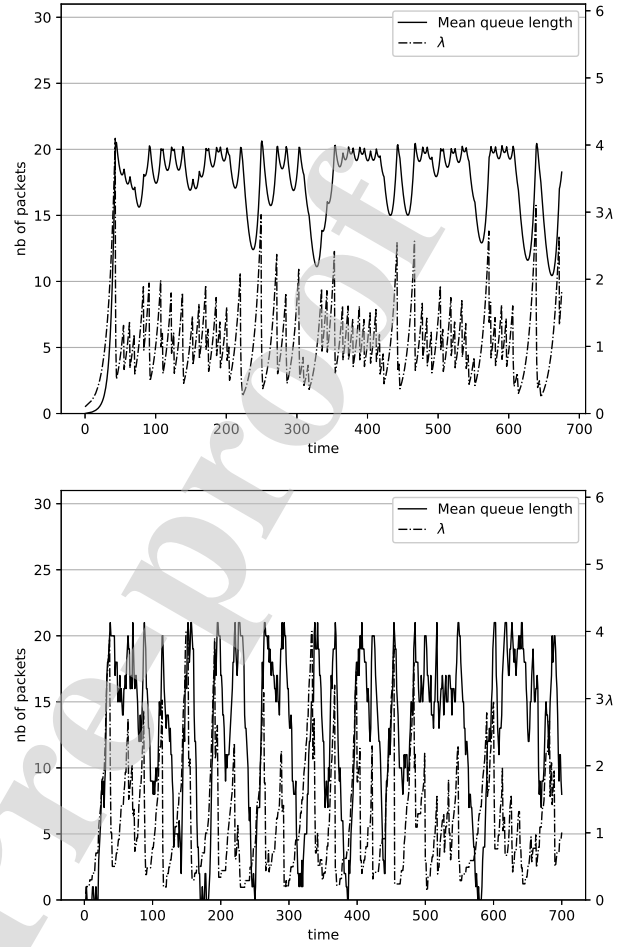
Figure 1: The router mean queue length and source intensity, diffusion approximation (upper), simulation (bottom), RED algorithm.

	K_p	K_i	η	setpoint
1	0.0001	0.0004	-1.2	10
2	0.0001	0.0004	-1.0	10
3	0.0001	0.0014	-0.8	10
4	0.0001	0.0040	-0.4	10

Table 1
PI^η controllers coefficients.

single experiment is shown in figure 1. The calculation time for a single simulation experiment does not exceed one second. In the case of diffusion calculations it takes a few minutes. Each figure presents two plots: mean queue length with y axis on the left and source intensity with y axis on the right. The queue occupancy varies according to the change in the intensity of the source (related to the TCP mechanism).

On the first stage of the experiment, we considered the RED algorithm as AQM. Sample results of changes in the traffic intensity and queue occupancy are presented in figure 2. Table 2 presents the detailed results for all performed experiments.

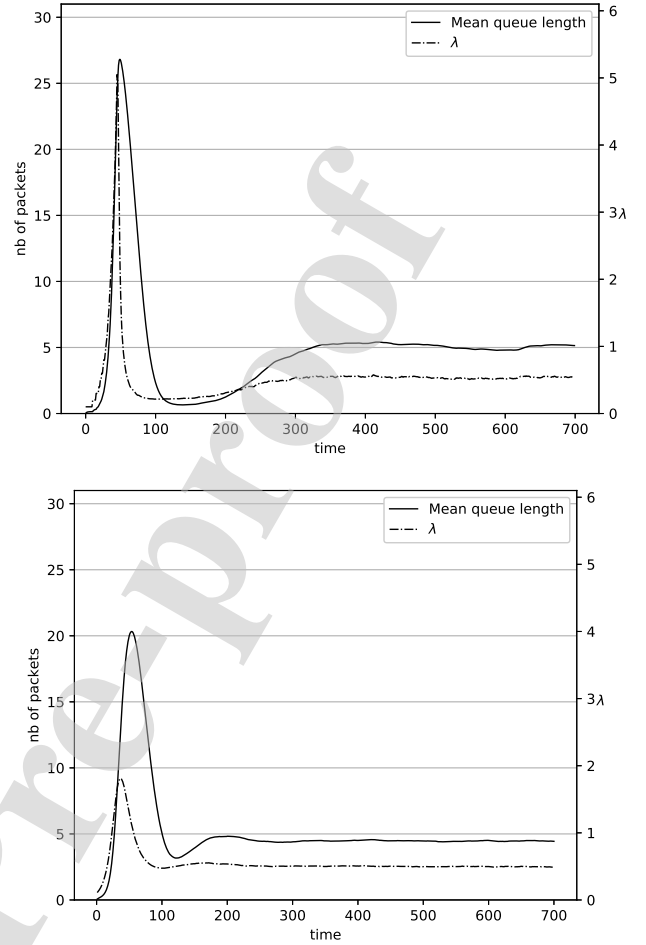Figure 2 shows that the mean queue length reaches the
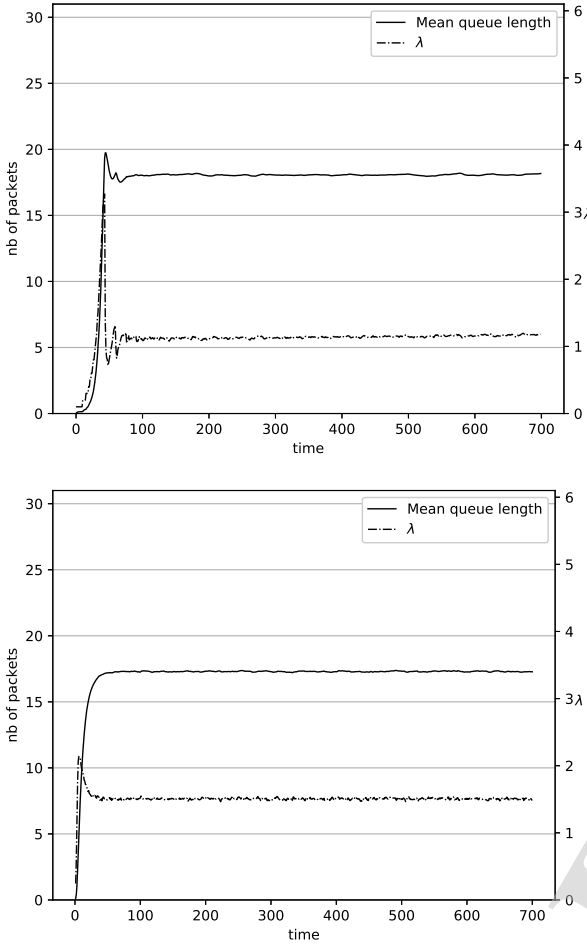
Figure 2: The router mean queue length and source intensity, diffusion approximation (upper), simulation (bottom), RED algorithm.

Figure 3: The router mean queue length and source intensity, diffusion approximation (upper), simulation (bottom), 1st set of PI^η parameters.

value of 17 packets. This is in line with the results presented in tab. 2. You can see in the same figure that congestion window control mechanism sets the source intensity at around 1,2.

In the next phase of the experiments, we evaluate the AQM mechanism with packet dropping probability function based on PI^η. We consider four different controllers. The PI^η controller with parameter $\eta = -1$ denotes the classic PI controller. The table 1 presents their parameters. The influence of the parameters on the efficiency of AQM queue was discussed in [62, 66, 67]. The proposed controllers differ in the effectiveness of maintaining the desired queue size.

Figures 3, 4, 5, 6 present how the intensity of the source evaluates over the time and how it affects the queue occupancy.

The first controller (2nd row in the tab. 1) is the most effective (we call it the most powerful) but, it rejects a large number of packets. In the case of this controller the mean queue length reaches the value of 5 packets (Fig. 3). The fourth type of controller (5th row in the tab. 1) has inverse properties. It rejects a smaller number of packets but its

AQM	Diffusion	Simulation
RED	18.067	17.106
PI^η 1	5.100	5.251
PI^η 2	6.122	6.302
PI^η 3	7.780	7.123
PI^η 4	10.431	10.075

Table 2
PI^η obtained average queue lengths.

mean queue length reaches a bit higher value, about 10 packets. The characteristics of the second and third controller (3rd and 4th rows in the tab. 1) are in-between the properties of the first and the fourth one. The mean queue lengths for these controllers reach accordingly the values of 6 and 7 packets.

The detailed results confirm the outcomes presented in figures. The average queue lengths in steady-state (fig. 2, 3, 4, 5, 6.) are identical to those presented in the table 2.

Additionally, for all AQM mechanisms (including the RED algorithm) differences in results obtained by diffusion approximation and simulations are negligible.
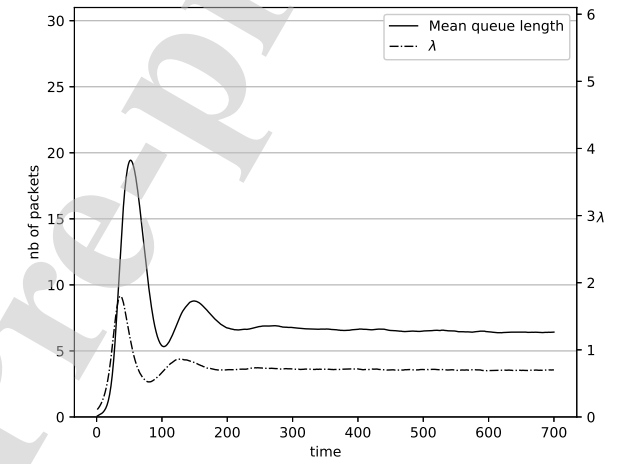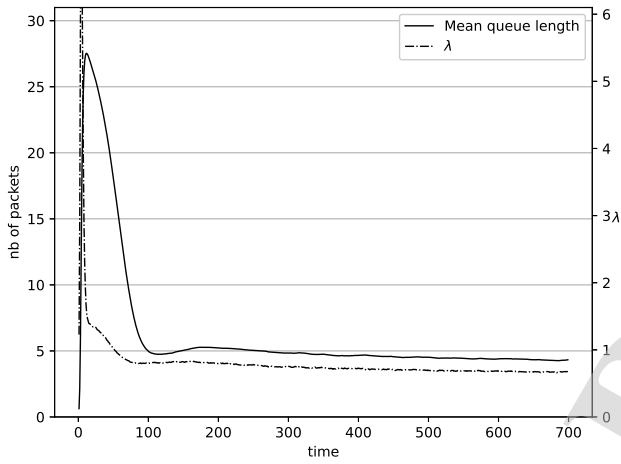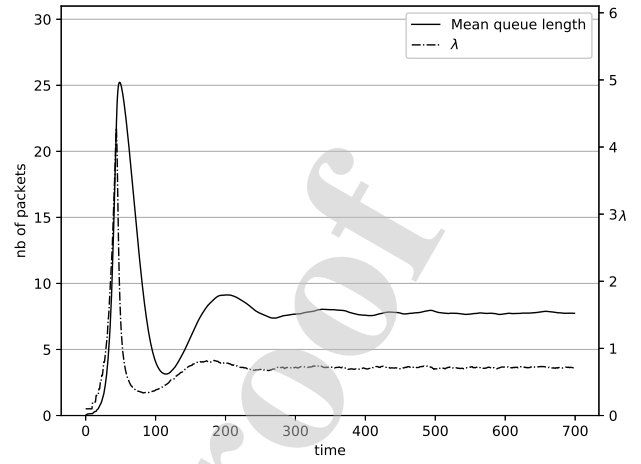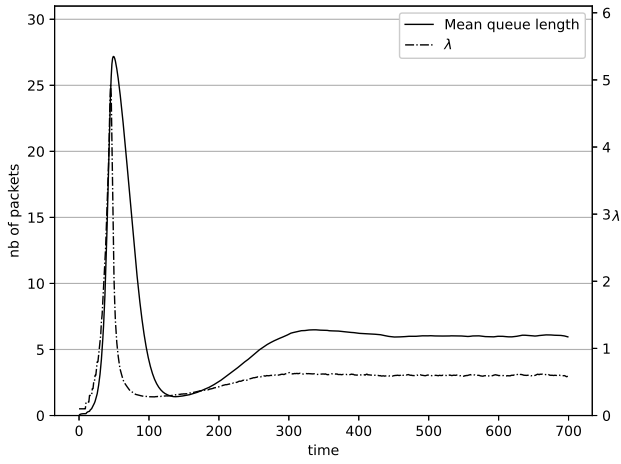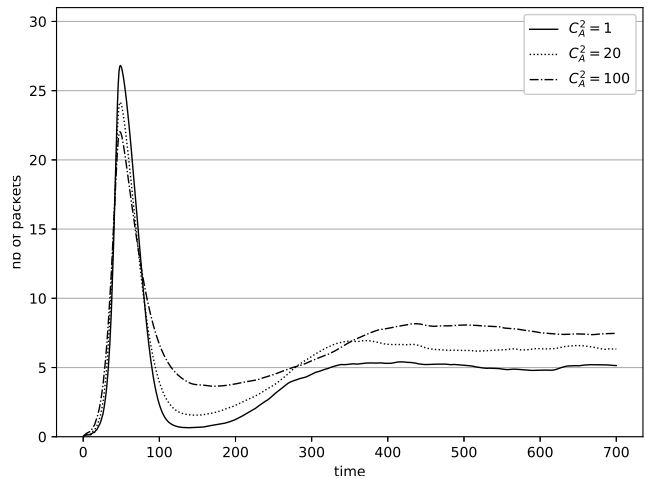
Figure 4: The router mean queue length and source intensity, diffusion approximation (upper), simulation (bottom), 2nd set of PI^{η} parameters.

Figure 5: The router mean queue length and source intensity, diffusion approximation (upper), simulation (bottom), 3rd set of PI^{η} parameters.

The influence of controller parameters on the behaviour of the AQM queue is visible. The average queue length decreases (tab. 2) with increasing controller fractional order (tab. 1).

The advantage of diffusion approximation is the ability to consider two first moments of traffic (intensity and variation), instead of only one in case of more foreign full used fluid flow approximation. Figure 7 presents the influence of the traffic source variation on queue behaviour. The impact is relatively low. We think that it is due to the performance of TCP protocol and its mechanism of adapting the transmission speed to the network capabilities.

Figure 7: The router mean queue length, G/M/1/30 queue, impact of traffic variation, 1st set of PI^{η} parameters.
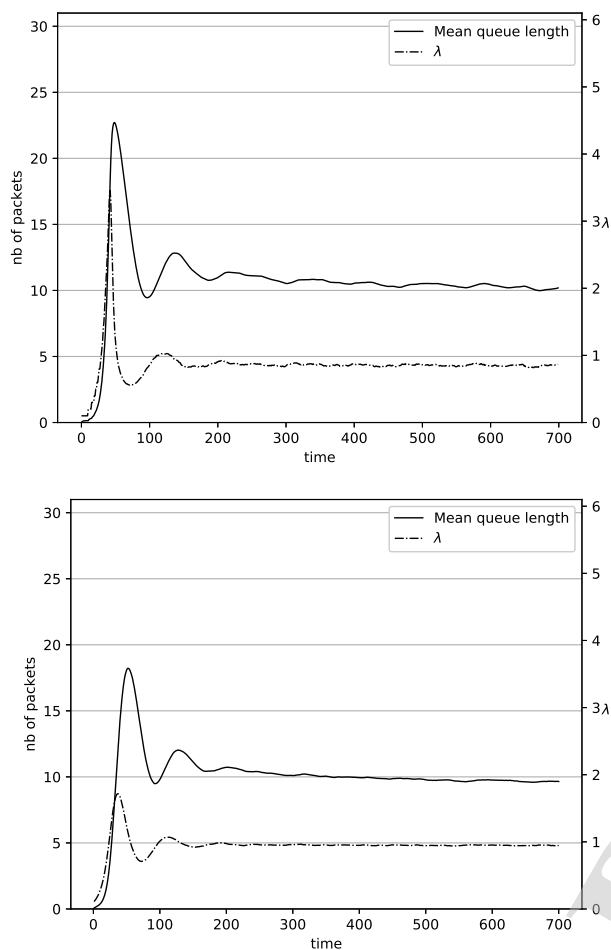
Figure 6: The router mean queue length and source intensity, diffusion approximation (upper), simulation (bottom), 4th set of PI^η parameters.

6. Conclusions

This article proposes a model combining the diffusion approximation with discrete event simulation. The approximation is used to calculate the size of the queue. The advantage of this approach is a natural description of transient states for any interarrival and service time distributions. The simulation part decides packet rejection following the AQM rules. This decision affects the source intensity of λ in the diffusion model. We consider the RED, PI and PI^η algorithms.

We compare the results obtained from the proposed mixed diffusion-simulation method with the results obtained by simulation. In both cases, the results are consistent. In the case of the PI^η controller, the obtained average queue sizes are also correct and dependent on the controller's power. An increase in controller power reduces the average queue size.

Our study shows the advantages of using the PI^η controller as an AQM mechanism. The use of the Fractional Calculus in the proposed mechanism allows for more precise regulation of the queue length in the communication node. The PI^η controller used as the AQM mechanism requires

more computing power and more memory than the RED algorithm. However, the computational cost of the proposed mechanism becomes less important with increasing routers computing power.

The results presented here also confirm the correctness of the model introduced in our previous article [32]. A single diffusion experiment takes several minutes. This is a disadvantage compared to other methods (e.g. fluid-flow aproximation [68]). But unlike this method, diffusion approximation offers more accurate results.

In our future work, we will focus on the diffusion models reflecting real Internet traffic (increased number of transmitters and the presence of both TCP and UDP streams).

Declaration of competing interest

No author associated with this paper has disclosed any potential or pertinent conflicts which may be perceived to have an impending conflict with this work.

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Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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