

Quantum implementation of Parrondo's paradox

Jarosław Miszczak, Piotr Gawron

Institute of Theoretical and Applied Informatics,
Polish Academy of Sciences, Gliwice, POLAND

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Overview

1. Motivation and goals
2. What is Parrondo paradox?
3. Grover's algorithm as an example
4. Review of classical and quantum implementations
5. Description of proposed implementation
6. Simulation results

Motivation and goals

- ~> Implementation of Parondo's paradox in Quantum Computer Language (QCL)
 - Since we are using simulation, it is very important to use as few qubits as possible.
- ~> Development of new quantum algorithms and protocols
 - Grover's algorithm is an example of Parondo paradox

Parrondo's paradox (1)

In 1996 Spanish mathematician J. M. R. Parrondo in his work „*How to cheat a bad mathematician*” proposed how to combine two games with negative expected value of the outcome (“losing game”) to obtain new game with positive expected value of the outcome (“winning game”).

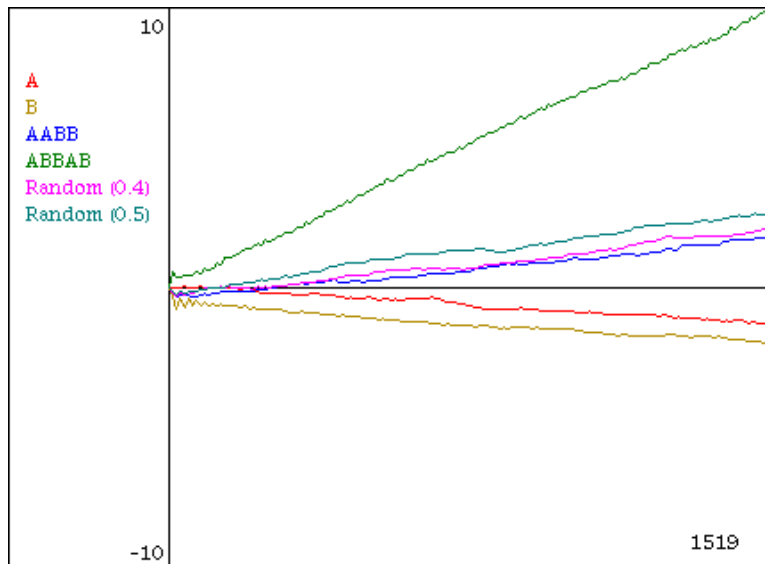
Parrondo's paradox consists of a sequence of games, where each game can be interpreted as a toss of an asymmetrical coin. Every success means that the player gains one Euro, every loss means that the player loses one Euro.

Parrondo's paradox (2)

- Game **A** consist of one sub-game \mathbb{A} with probability of winning $1/2 - \epsilon$.
- Game **B** consists of two sub-games. It depends on the amount of capital accumulated by player which one is played. If his capital is a multiple of three, the player tosses coin B_1 , which has probability of wining $1/10 - \epsilon$, otherwise the player tosses coin B_2 which has probability of wining $3/4 - \epsilon$.

Originally $\epsilon = 0.005$, but generally it can be any small real number.

Some examples – simulation of classical paradox



Parametry:

$$\epsilon = 0.005$$

$$P = 0.5$$

$$P_1 = 0.1$$

$$P_2 = 0.75$$

ilość gier: 150

prób 1500

Comments on results

- strategy "always **A**" give negative expected gain (loosing game),
- strategy "always **B**" give negative expected gain (loosing game B_1 is played more often),
- strategies "AABB" i "ABBAB" give relatively high expected gain,
- random strategy *i.e.* "choose **A** or **B**" also give relatively high expected gain.

Quantum and classical realizations

– capital-dependent

★ Original version by Parrondo.

★ Meyer, Blummer (J. Stat. Phys. **107**, 1-2 (2002)) – using quantum random walk

★ Prproposed implementation

– History-dependent

★ Parrondo, Harmer, Abbot (PRL **85**, 24 (2000)) – game **B** consists of 4 elementary games (or sub-games) and realization of them depends on last 4 outcomes

★ Flitney, Ng, Abbott (Physica A **314** (2002)) – coin tossing is replaced by rotation, **Main disadvantage: needs many qubits!**

Grover's algorithm

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Proposed realization (1)

We propose new implementation of the capital-dependent quantum Parrondo paradox. In our scheme we are using following registers

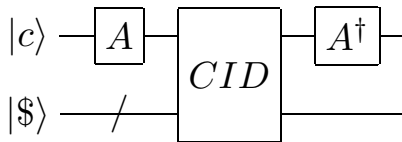
- $|c\rangle$: one-qubit register representing the coin,
- $|\$\rangle$: n -qubit register storing player's capital,
- $|o\rangle$: three-qubit auxiliary register.

Main goal of this realization is to obtain behaviour typical for Parrondo's paradox using small number of qubits.

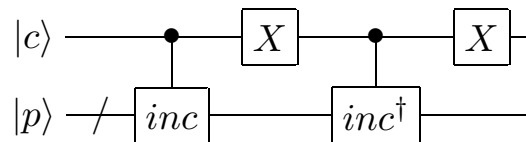
Proposed realization (2)

Gra **A** jest realizowana jako bramka jednpqubitowa. Dodatkowo wykorzystywana jest operacja CID, która zmienia stan rejestru w zależności od wyniku gry.

Gra A



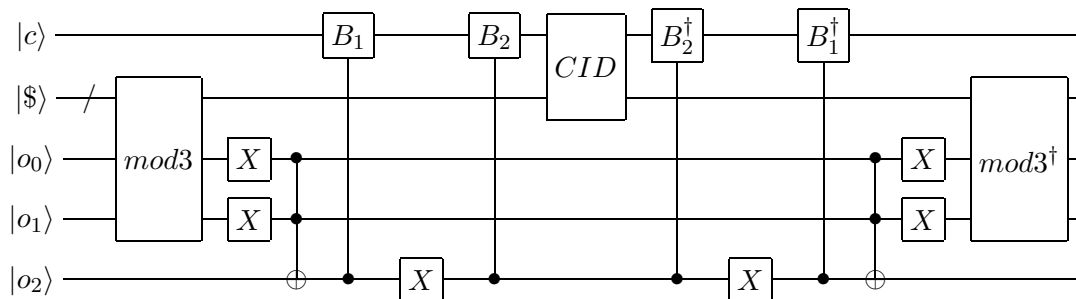
Conditional increment/decrement (CID)



Proponowana implementacja paradoksu (3)

Do realizacji gry **B** potrzebne są dwie bramki realizujące gry B_1 i B_2 oraz operacja $mod3$, realizująca sprawdzanie podzielności kapitału przez 3.

Gra B



Proponowana implementacja paradoksu (4)

Konstrukcja schematu

1. Przygotowanie rejestru $|c\rangle$ w stanie $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$.
2. Przygotowanie rejestru $|\$ \rangle$ w stanie $|(2^{(n-1)} + \text{offset})\rangle$, gdzie *offset* jest małą liczbą całkowitą. Takie ustalenie rejestru jest niezbędne aby uniknąć arytmetycznego przepełnienia rejestru.
3. Wyzerowanie rejestru pomocniczego $|o_1, o_2, o_3\rangle$: $|000\rangle$.
4. Działanie bramkami A i B w pewnym wybranym porządku \mathbb{S} (strategii).

Proponowana implementacja paradoksu (5)

Pomiar

Symulacja pozwala na wgląd w wektor stanu rejestru. Korzystając z tej własności wartość oczekiwana jest liczona jako średnia wartość obserwabli σ_z w stanie $|\$\rangle\langle\$| = Tr_{|c\rangle\otimes|o\rangle}(|c, \$, o\rangle\langle c, \$, o|)$ otrzymanym po odrzuceniu qubitów monety i pomocniczych:

$$\langle\$| = Tr(\sigma_z|\$\rangle\langle\$|). \quad (1)$$

Symulacja

Zaproponowany schemat został zaimplementowany w języku QCL.

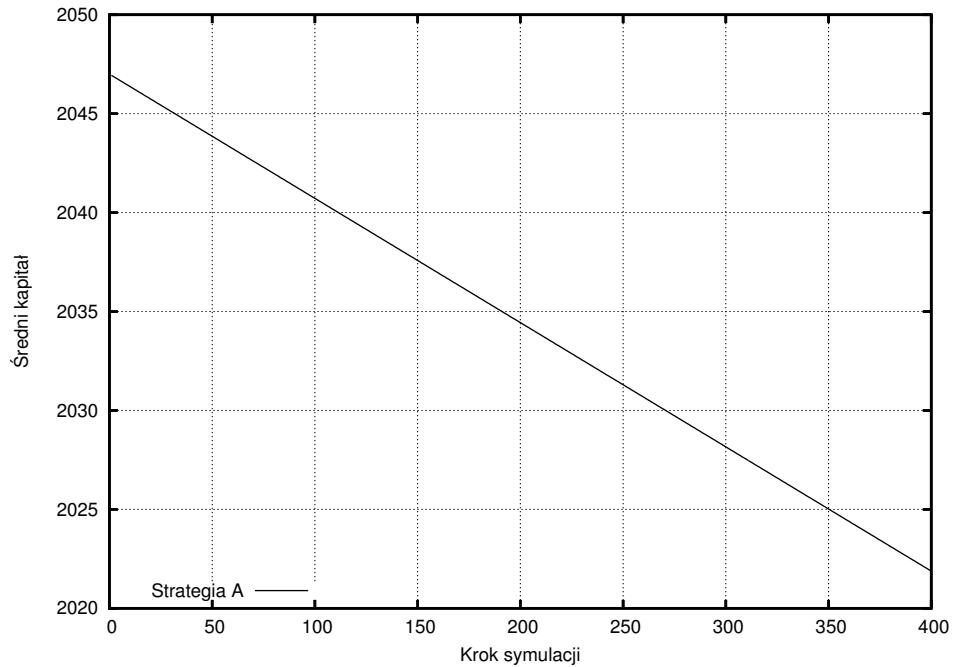
Parametry

δ_A	α_A	β_A	θ_A
0	1	0	$2(\frac{\pi}{2} + 0.01)$
δ_{B_1}	α_{B_1}	β_{B_1}	θ_{B_1}
0	1	0	$2(\frac{\pi}{10} + 0.01)$
δ_{B_2}	α_{B_2}	β_{B_2}	θ_{B_2}
0	1	0	$2(\frac{3\pi}{4} + 0.01)$

$$\text{offset} = -1$$

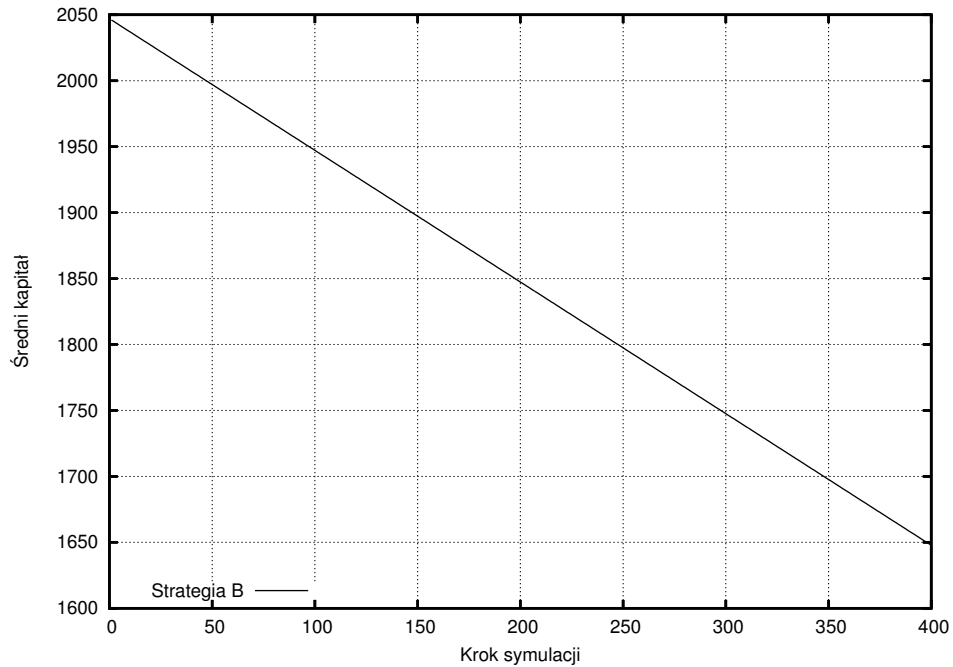
$$\begin{aligned}
 Ph(\xi) &= \begin{pmatrix} e^{i\xi} & 0 \\ 0 & e^{i\xi} \end{pmatrix}, \\
 R_y(\xi) &= \begin{pmatrix} \cos(\frac{\xi}{2}) & -\sin(\frac{\xi}{2}) \\ \sin(\frac{\xi}{2}) & \cos(\frac{\xi}{2}) \end{pmatrix}, \\
 R_z(\xi) &= \begin{pmatrix} e^{-i(\frac{\xi}{2})} & 0 \\ 0 & e^{i(\frac{\xi}{2})} \end{pmatrix} \\
 G(\delta_G, \alpha_G, \theta_G, \beta_G) &= \\
 &= R_z(\beta_G)R_y(\theta_G)R_z(\alpha_G)Ph(\delta_G)
 \end{aligned}$$

Przykładowe wyniki (1)



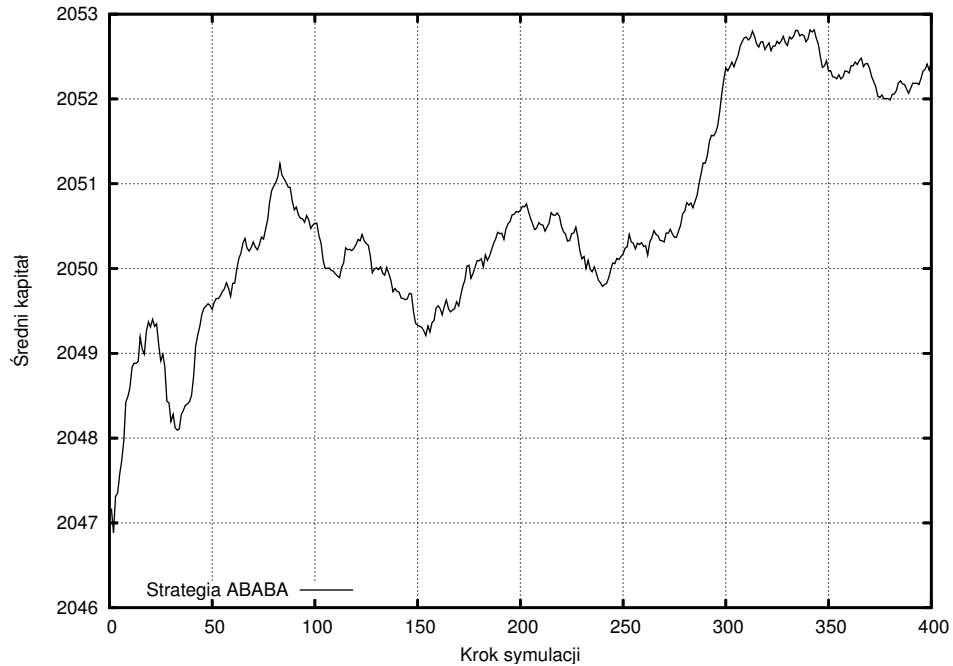
Strategia A^{400}

Przykładowe wyniki (2)



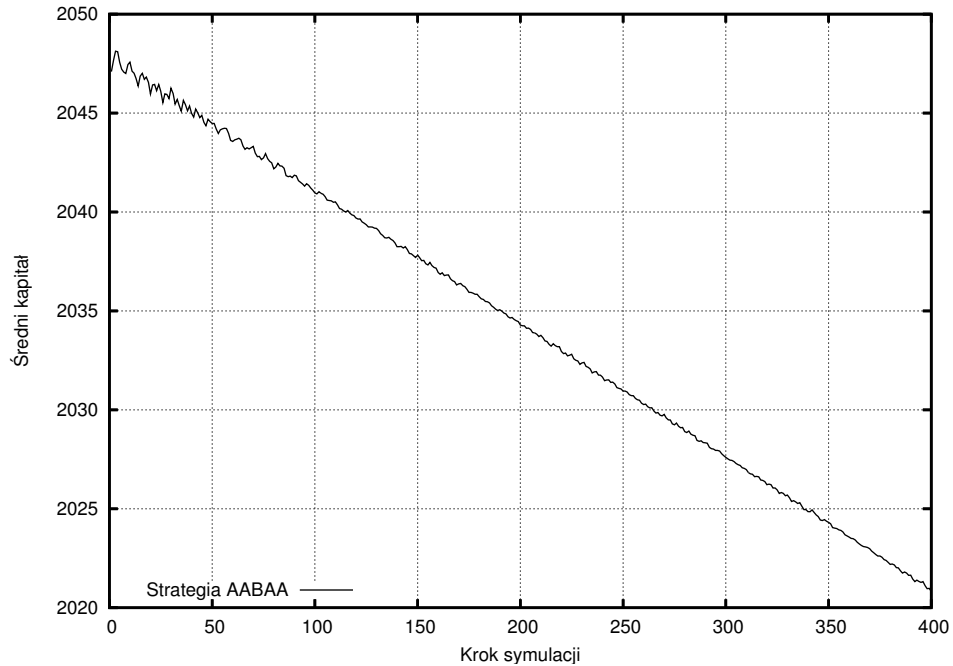
Strategia B^{400}

Przykładowe wyniki (3)



Strategia $(ABABA)^{400}$

Przykładowe wyniki (4)



Strategia $(AABAA)^{400}$

Podsumowanie i dalsze prace

- Zaproponowana implementacja potrzebuje tylko $O(\log(n))$ qubitów dla n -kroków (gier).
- Zostały znalezione „paradoksalne” strategie dla zaproponowanego schematu.
- ↪ Istnieje potrzeba przebadania tego schematu w sposób analityczny.
- ↪ Czy można wykorzystać paradoks Parrondo do walki z dekoherencją?

References

- [1] G. P. Harmer, D. Abbott, *Parrondo's paradox*, Statistical Science, Vol. 14, No. 2, (1999), pp. 206-213
- [2] J. M. R. Parrondo, G. P. Harmer, D. Abbott, *New Paradoxical Games Based on Brownian Ratchets*, Phys. Rev. Lett. 85, 5226-5229 (2000).
- [3] A. P. Flitney, D. Abbot, *Quantum models of Parrondo's games*, Physica A, 324 (2003), pp. 152-156.
- [4] J. A. Mischczak, P. Gawron, *Quantum Implementation of Parrondo Paradox*, quant-ph/0502185. Przesłane do Fluctuation and Noise Letters.
- [5] Adelaide Parrondo's Paradox Group,
<http://www.eleceng.adelaide.edu.au/Groups/parrondo/>



Questions...

<http://www.iitis.gliwice.pl/zksi/>